

# Effects of static and dynamic disorder on the performance of neural automata

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## Abstract

We report on both analytical and numerical results concerning stochastic Hopfield-like neural automata exhibiting the following (biologically inspired) features: (1) Neurons and synapses evolve in time as in contact with respective baths at different temperatures; (2) the connectivity between neurons may be tuned from full connection to high random dilution, or to the case of networks with the small-world property and/or scale-free architecture; and (3) there is synaptic kinetics simulating repeated scanning of the stored patterns. Although these features may apparently result in additional disorder, the model exhibits, for a wide range of parameter values, an extraordinary computational performance, and some of the qualitative behaviors observed in natural systems. In particular, we illustrate here very efficient and robust associative memory, and jumping between pattern attractors.

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## 1. The model and its motivation

The field of *neural automata*, or cellular automata [1], which are biologically inspired and aimed at solving open issues in neuroscience, may be said to initiate with the Hopfield model [2]. This illustrates—on a very simple and, consequently, rather unrealistic grounds—the property of associative memory, a task for which natural systems remain far from unbeaten by digital computers. Besides many other related studies [3], the Hopfield model has been generalized along three interesting lines. That is, it has been shown that one may enhance the computational abilities of a Hopfield-like system by allowing for dynamic—instead of quenched—synapses [4], by implementing dynamics in the computer via parallel—instead of sequential—updating [5,6], and by considering scale-free—instead of extremely connected—net-

work topologies [7,8]. In this paper we report on some recent efforts along these lines. We describe a stochastic neural automata scheme that allows for explicitly studying the influence of the model details on the resulting computational performance. In particular, we evaluate the effects of disorder perturbing both (statically) the network architecture and (dynamically) the synaptic strengths. The importance of these two details on the functioning of natural neural systems has been recently recognized (e.g., in Refs. [9,10], respectively). We conclude that the computational performance may indeed be importantly enhanced by appropriately tuning these details. This is illustrated here with some specific cases; the range of validity of our main results is known to go beyond these examples, however.

Consider a network with a binary neuron variable,  $s_i = \pm 1$ , at each node  $i = 1, \dots, N$ . Assume pair synapses of weight  $w_{ij}$  connecting the neurons at  $i$  and  $j$ , and define the degree  $k_i$  of node  $i$  as the number of links from  $i$  to any other node. A scale-free network [11] has power-law distributed degrees  $P(k) \sim k^{-\gamma}$ , while  $P(k) \sim \delta(k-N)$ , for the fully connected case (each neuron links to any other one in the network), and  $k$  has

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a mean value  $\zeta N$ ,  $0 < \zeta < 1$ , for the randomly  $\zeta$ -diluted case. The synaptic links are known to serve to store information. In the standard Hopfield model, for instance, one assumes  $M$  patterns consisting of binary variables,  $\xi_i^\mu = \pm 1$ ,  $\mu = 1, \dots, M$ , and the weights are then set according to some *learning rule*. A familiar instance is the Hebb's rule for which the synapses take values  $w_{ij} \propto \sum_{\mu} \xi_i^\mu \xi_j^\mu$ .

The system configuration at time  $t$  involves the sets  $\mathbf{S} \equiv \{s_i\}$  and  $\mathbf{W} \equiv \{w_{ij}\}$  which together determine the value of some *energy function*. To be specific, consider this to be  $H(\mathbf{S}, \mathbf{W}; t) = -1/2 \sum_i \sum_j^{(i)} w_{ij}(t) s_i s_j$  where the second sum goes over all nodes connecting to  $i$ . That is, we are assuming that there is an energy function with the Ising structure *at each time  $t$* , and that synapses vary with  $t$ . For simplicity, we further assume that  $w_{ij}(t) = \bar{w}_{ij}[\mu(t)] = \sigma N^{-1} \xi_i^{\mu(t)} \xi_j^{\mu(t)}$ , where  $\sigma$  is a normalization constant. This amounts to setting each connection at time  $t$  equal to the value obtained from the  $\mu$ th pattern via a *learning rule*. And our choice for this (see what follows) is such that, after time averaging, it results in the Hebb's rule<sup>1</sup> (i.e., we average over all patterns with the same weight). A main fact is that, consistently with the empirical observation that memory is a global dynamic phenomenon, we are assuming that all local synapses are set at each time by a given pattern, which is chosen according to some dynamics to be determined next.

One is mainly interested in  $\mathbf{m} \equiv \{m^\mu; \mu = 1, \dots, M\}$ , where  $m^\mu = m^\mu(\mathbf{S})$  is the overlap between the current state  $\mathbf{S}$  and pattern  $\mu$ . We assume [12] (see also Cortes et al., in preparation, 2004) that  $(\mathbf{S}, \mu)$  evolves in discrete time according to a transition rate  $T$  that may be decomposed as:  $T'_\mu[\mathbf{S}' \rightarrow \mathbf{S}] \times T_\mu[\mu' \rightarrow \mu]$ . Here,  $T_\mu$  is taken as a product of  $N$  terms  $\Psi[\beta_0 \Delta H(s'_i \rightarrow s_i = \pm s'_i)]$  (as corresponds to the neuron configuration varying by parallel updating or *little dynamics*), and  $T_\mu = \Psi[\beta_1 \Delta H(\mu' \rightarrow \mu)]$ . The function  $\Psi(X)$  is arbitrary (e.g., of the Glauber type [13]) and each transition rate involves both an inverse temperature  $\beta$  of the bath inducing its own stochasticity and the corresponding change  $\Delta H$  of the chosen energy function.

The fact that the neuron activities and the synaptic intensities are stochastically driven by different parameters,  $\beta_0^{-1}$  and  $\beta_1^{-1}$ , respectively—which one may imagine as the temperatures of two different baths in contact with each set of degrees of freedom—endows the neural automata with a varied and interesting behavior. This is related to the fact that the competition between baths impedes the system from reaching thermal equilibrium; in general, the asymptotic state is instead a nonequilibrium steady state, which is more realistic than an equilibrium one [13]. Although the bath competition is, to the best of our knowledge, a novel feature within the present context, our model has some close antecedents [14–16]; these, however, mainly concern

sequential updating, which in general results in a less efficient mechanism (Cortes et al., in preparation, 2004). Furthermore, our model describes the changes in the two subsystems on the same time scale, which is an interesting general situation with well-defined limits. That is,  $\beta_1 \rightarrow \infty$  corresponds to the freezing of synapses [2]: large  $\beta_1$  is for slow synaptic kinetics [14], and small  $\beta_1$  describes extreme synaptic activity [15].

This neural automata is also simple and versatile enough to allow for analyzing different network architectures. With this aim, we (i) first generated a network starting with  $\eta_0$  nodes and adding  $\eta \leq \eta_0$  links at consecutive time steps with the preferential attachment rule. This, resulting in  $\eta(N - \eta_0)$  links for  $N$  nodes, will be called the scale-free (Barabasi–Albert) network (SFN) [11]. (ii) A fully connected network (FCN) with  $N$  nodes has  $N^2$  links, which is not realistic for a neural system. Instead, (iii) a meaningful alternative to the SFN case is the (highly) diluted network (DN) obtained by randomly suppressing links in the FCN until only  $\eta(N - \eta_0)$  is left. In order to have a true SFN with the *small-world* property (i.e., most of the nodes exhibiting small connectivity and a few *hubs* having its connectivity comparable to the network size  $N$ , one needs to restrict to  $\eta \ll N$ .

## 2. Scale-free topology of synapses

The small-world property, which implies that the average path length between any two nodes is very small compared to the network typical length, is a suitable feature for an efficient neural system. In fact, this property has been reported to hold in many natural systems [11,17], including growing cultured neurons [9], and it was shown that the SFN can store and retrieve a given number of patterns with a lower computer memory cost than the FCN [7]. We shall briefly illustrate here (see also Ref. [8]) that the scale-free architecture may indeed enhance both the network associative performance and its robustness against thermal noise perturbations.

The graphs in Fig. 1 show the dependence of the stationary overlap  $m^\mu$  on the neuron temperature,  $T \equiv \beta_0^{-1}$ , for cases SFN and DN with  $\eta = 3$  and  $M = 1$ . Fig. 1 illustrates that the SFN makes indeed a better job for retrieving of information as compared to DN at sufficiently high temperatures, which is the relevant case for practical purposes. As  $M$  is increased, the performance tends to deteriorate in both cases. However, one then observes an intriguing behavior of the SFN at finite temperature. That is, there is a definite tendency of the *hubs*, namely, nodes with  $k \geq k_0(M, \beta_0, \beta_1)$ , to concentrate most of the system activity concerning the retrieval process.

In order to illustrate the above property, we computed the mean *local overlap* associated to a node of degree  $k$ ,  $m(k)$ . The resulting graphs in Fig. 2 clearly depict that  $m(k) \rightarrow 1$  as  $k$  increases at any temperature (large fluctua-

<sup>1</sup> It is worth remarking that this and several other details here are rather arbitrary. They mainly intend to be an easy illustration of the relevant result, and can generally be waived or modified—sometimes rather straightforwardly.

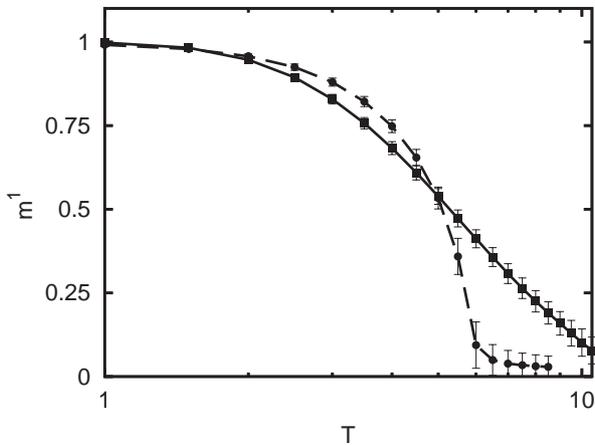


Fig. 1. Stationary overlap  $m^1$  versus the logarithm of the neuron temperature (arbitrary units), averaged over 100 histories for  $N=1600$  neurons,  $M=1$ , and  $\eta=\eta_0=3$ , for the scale-free network (squares) and for the diluted network (circles). This illustrates a better performance of the former in general for practical purposes.

tions simply reflect that the number of hubs is small for the network size used). This indicates how the hubs, the more the higher its connectivity degree, tend to become robust references for the process of associative memory. The state of *boundary nodes*, on the contrary, shows a poor correlation with the relevant stored pattern. This is in agreement with a previous observation at zero (neuron) temperature [18]. The above results triggered our interest in networks in which the connectivity degree is power-law distributed,  $P(k)\sim k^{-\gamma}$ . An issue is the possible influence of the parameter  $\gamma$  on the system performance. In fact, the number of highly connected nodes tends to sharply decrease as  $\gamma$  is increased, and it ensues  $\gamma\lesssim 2$  as a convenient range. Further study of this will be reported elsewhere [8].

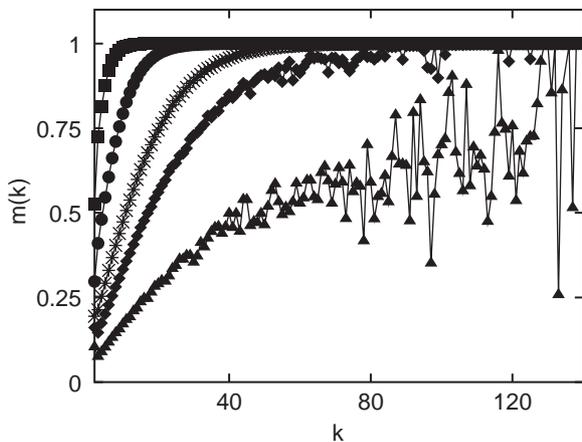


Fig. 2. The *local overlap*, as defined in the main text, versus the connectivity degree  $k$  for different  $T$ , increasing from top to bottom, for the scale-free network in Fig. 1. This illustrates a tendency of the *hubs* to control the retrieval of information.

### 3. Escaping from the attractor

Time behavior is also intriguing as illustrated, for instance, by the FCN architecture, which is more amenable to analysis. Fig. 3 depicts the various *phases* exhibited by the system as the parameters  $\beta_0$  and  $\beta_1$  are changed. There is a *ferromagnetic* (F) phase in which the system shows associative memory, and a *paramagnetic* (P) phase—the upper stripe—lacking this property. This is familiar from the Hopfield model. A first novelty is that no *mixed states* [15] occur, which is computationally convenient. In addition, the nonequilibrium phase diagram in Fig. 3 depicts a region between the P and F phases that exhibits emergent dynamic behavior confirming (and extending) a result in Ref. [10]. That is, the system in this region has *dynamic* associative memory, namely, after a transient time in which one of the stored patterns is recovered, the system jumps to one of the other possible attractors, and keeps indefinitely doing so. The nature of this jumping deserves a comment. Simulations uncover that there is some nontrivial structure of time correlations. This is revealed by monitoring the time  $\tau_{v,\zeta}$  the system stays in attractor  $v$  before escaping to another one, say  $\zeta$ . Within the largest jumping region, O(II),  $\tau_{v,\zeta}$  happens to be practically independent of both  $v$  and  $\zeta$ . That is, the system then stays the same amount of time wandering in each attractor. However, the probability of jumping between patterns depends on the activities, and nontrivial time correlations develop as the neuron temperature is lowered, namely, in region O(I), where one even observes that  $\tau_{v,\zeta}\neq\tau_{\zeta,v}$ . The behavior in O(I) suggests using our algorithm to code spatial-temporal information [19].

Summing up, we have illustrated several aspects of the behavior of Hopfield-like neural automata. This consists of neurons and synapses that evolve on the same time scale but subject to different thermal noises. Furthermore, different

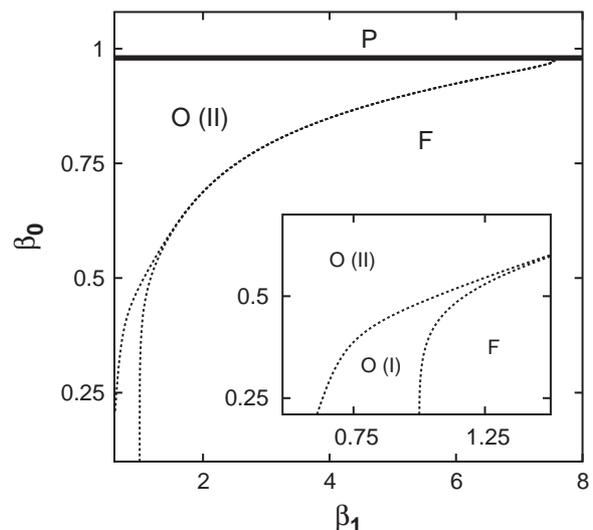


Fig. 3. Phase diagram for the FCN automata depicting three coexistence curves that define several phases (see the main text).

network architectures have been considered. It follows that a power-law topology, which is known to characterize many natural, including neural, systems is advantageous compared to the corresponding diluted network. We also find definite evidence that *hubs* (i.e., the few most highly connected nodes in a scale-free architecture) play a fundamental role in making the retrieval of information more robust and efficient. These findings, whose validity in natural systems is to be checked, suggest paths for a convenient design of artificial systems. We have also demonstrated that, for appropriate parameter values, neural automata perform much more efficiently when one lets the subsystem of synapses to constantly and coherently visit all the stored patterns in a convenient way. This is fully consistent with two main empirical observations, namely, that memory is a global dynamic phenomenon, and that oscillations are essential to cortex functions. The neural automata has also been shown to exhibit in some situations spatial-temporal attractors, which may be relevant to simple olfactory processing, for instance.

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